FUNCIONES TRIGONOMÉTRICAS HIPERBÓLICAS senh( )

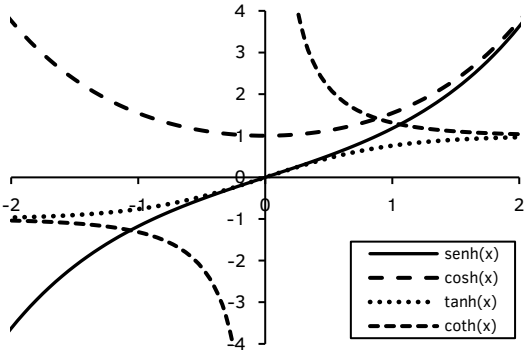
Las funciones trigonométricas hiperbólicas son combinaciones especiales de funciones exponenciales, que aparecen en la solución de algunas ecuaciones diferenciales. Se les llama así porque tienen algunas características similares a las funciones trigonométricas (circulares).

Definiciones

$$\operatorname{senh} x \equiv \frac{e^x - e^{-x}}{2} \qquad \cosh x \equiv \frac{e^x + e^{-x}}{2}$$

$$\tanh x \equiv \frac{\operatorname{senh} x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \coth x \equiv \frac{1}{\tanh x}$$

Gráfica



Valores límite

	$x \to 0$	$x \to -\infty$	$x \to \infty$
$\operatorname{senh} x =$	0	$-\infty$	∞
$\cosh x =$	1	∞	∞
$\tanh x =$	0	-1	1
$\coth x =$	$\pm\infty$	-1	1

Relaciones mutuas

$$\operatorname{senh} x = \sqrt{\cosh^2 x - 1} = \frac{\tanh x}{\sqrt{1 - \tanh^2 x}} = \frac{1}{\sqrt{\coth^2 x - 1}}$$

$$\cosh x = \sqrt{\operatorname{senh}^2 x + 1} = \frac{1}{\sqrt{1 - \tanh^2 x}} = \frac{\coth x}{\sqrt{\coth^2 x - 1}}$$

$$\tanh x = \frac{\sinh x}{\sqrt{\sinh^2 x + 1}} = \frac{\sqrt{\cosh^2 x - 1}}{\cosh x} = \frac{1}{\coth x}$$

Argumento negativo

$$\operatorname{senh}(-x) = -\operatorname{senh} x \qquad \cosh(-x) = \cosh x$$

$$\tanh(-x) = -\tanh x \qquad \coth(-x) = -\coth x$$

Relación con números complejos

$$\operatorname{senh}(iz) = i\operatorname{sen}(z) \qquad \cosh(iz) = \cos(z)$$

$$\operatorname{senh}(z) = -i\operatorname{sen}(z) \qquad \cosh(z) = \cos(iz)$$

Identidades

$$\cosh x + \sinh x = e^x \qquad \cosh x - \sinh x = e^{-x}$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \tanh x \coth x = 1$$

$$1 - \tanh^2 x = \frac{1}{\cosh^2 x} \qquad 1 - \coth^2 x = \frac{-1}{\sinh^2 x}$$

$$\operatorname{senh}(2x) = 2\operatorname{senh} x \cosh x \qquad \cosh(2x) = 2\cosh^2 x - 1$$

$$\operatorname{senh}^2\left(\frac{x}{2}\right) = \frac{\cosh x - 1}{2} \qquad \cosh^2\left(\frac{x}{2}\right) = \frac{\cosh x + 1}{2}$$

Teoremas de adición

$$\operatorname{senh}(A \pm B) = \operatorname{senh} A \cosh B \pm \cosh A \operatorname{senh} B$$

$$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$$

$$\tanh(A \pm B) = \frac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$$

$$\coth(A \pm B) = \frac{\coth A \coth B \pm 1}{\coth A \coth B}$$

Derivadas

$$\frac{d}{dx}\operatorname{senh} u = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx}\cosh u = \operatorname{senh} u \frac{du}{dx}$$

$$\frac{d}{dx}\tanh u = \left(1 - \tanh^2 u\right)\frac{du}{dx}$$

$$\frac{d}{dx}\coth u = \left(1 - \coth^2 u\right)\frac{du}{dx}$$

Integrales

$$\int \operatorname{senh} u\, du = \cosh u + C$$

$$\int \cosh u\, du = \operatorname{senh} u + C$$

$$\int \tanh u\, du = \ln(\cosh u) + C$$

$$\int \coth u\, du = \ln(\operatorname{senh} u) + C$$

Funciones hiperbólicas inversas

$$\operatorname{arcsenh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\operatorname{arccosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$$