Series de Fourier

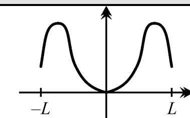
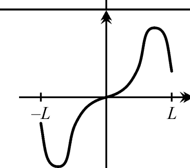
La serie de Fourier de una función $f(x)$ definida en $-L < x < L$ con periodo $p = 2L$ es:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \operatorname{sen}\frac{n\pi x}{L}\right)$$

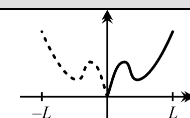
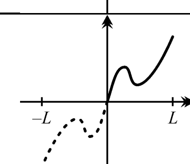
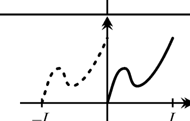
donde los coeficientes de Fourier están dados por:

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\operatorname{sen}\frac{n\pi x}{L}\,dx$$

Series de Fourier de funciones pares e impares

FUNCIÓN PAR	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$			
	$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$	$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$	$b_n = 0$	
FUNCIÓN IMPAR	$f(x) = \sum_{n=1}^{\infty} b_n \operatorname{sen}\frac{n\pi x}{L}$			
	$a_0 = 0$	$a_n = 0$	$b_n = \frac{2}{L}\int_0^L f(x)\operatorname{sen}\frac{n\pi x}{L}\,dx$	

Extensiones de medio intervalo de la serie de Fourier

EXTENSIÓN PAR	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$			
	$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$	$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$	$b_n = 0$	
EXTENSIÓN IMPAR	$f(x) = \sum_{n=1}^{\infty} b_n \operatorname{sen}\frac{n\pi x}{L}$			
	$a_0 = 0$	$a_n = 0$	$b_n = \frac{2}{L}\int_0^L f(x)\operatorname{sen}\frac{n\pi x}{L}\,dx$	
EXTENSIÓN PERIÓDICA	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L/2} + b_n \operatorname{sen}\frac{n\pi x}{L/2}\right)$			
	$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$	$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L/2}\,dx$	$b_n = \frac{2}{L}\int_0^L f(x)\operatorname{sen}\frac{n\pi x}{L/2}\,dx$	

Serie compleja de Fourier

$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{n\omega_0 x i}$ donde la frecuencia fundamental es $\omega_0 = \frac{2\pi}{p} = \frac{\pi}{L}$			
Coeficientes complejos	$c_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$	$c_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-n\omega_0 x i}\,dx$	$c_{-n} = \overline{c}_n$ (complejo conjugado)
Relación con coeficientes de la serie de Fourier	$c_0 = \frac{a_0}{2}$	$c_n = \frac{1}{2}(a_n - b_n i)$	$c_{-n} = \frac{1}{2}(a_n + b_n i)$

NOTA: Al efectuar la sumatoria en la serie compleja de Fourier, siempre tomar el mismo número de coeficientes positivos y negativos.

Algunas integrales útiles para series de Fourier

$$\int x\cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C$$

$$\int x\sin ax dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$$

$$\int x^2\cos ax dx = \frac{2x}{a^2}\cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\sin ax + C$$

$$\int x^2\sin ax dx = \frac{2x}{a^2}\sin ax - \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\cos ax + C$$

$$\int x^3\cos ax dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right)\cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3}\right)\sin ax + C$$

$$\int x^3\sin ax dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right)\sin ax - \left(\frac{x^3}{a} - \frac{6x}{a^3}\right)\cos ax + C$$

$$\int x^4\cos ax dx = \left(\frac{4x^3}{a^2} - \frac{24x}{a^4}\right)\cos ax + \left(\frac{x^4}{a} - \frac{12x^2}{a^3} + \frac{24}{a^5}\right)\sin ax + C$$

$$\int x^4\sin ax dx = \left(\frac{4x^3}{a^2} - \frac{24x}{a^4}\right)\sin ax - \left(\frac{x^4}{a} - \frac{12x^2}{a^3} + \frac{24}{a^5}\right)\cos ax + C$$

$$\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2 + b^2}\left(a\cos bu + b\sin bu\right) + C$$

$$\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2 + b^2}\left(a\sin bu - b\cos bu\right) + C$$

Otras relaciones útiles

$\cos n\pi = (-1)^n \qquad \sin n\pi = 0$