

Derivadas e Integrales

$$f(x) = \frac{d}{dx}F(x) \Leftrightarrow F(x) = \int f(x)dx + C$$

Donde sea apropiado, a, b, c, C y n son constantes, y F, f, u, v y w son funciones de x .

Derivadas básicas

$$1. \frac{df}{dx} \equiv \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$2. \frac{d}{dx}(c) = 0$$

$$3. \frac{d}{dx}(x) = 1$$

$$4. \frac{d}{dx}(cx) = c$$

$$5. \frac{d}{dx}(x^n) = nx^{n-1}$$

$$6. \frac{d}{dx}(cu) = c \frac{du}{dx}$$

$$7. \frac{d}{dx}(u \pm v \pm w \dots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \dots$$

$$8. \frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$$

$$9. \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$10. \frac{d}{dx}(uvw) = uv \frac{dw}{dx} + uw \frac{dv}{dx} + vw \frac{du}{dx}$$

$$11. \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$$

$$12. \frac{dF}{dx} = \frac{dF}{du} \frac{du}{dx} \quad (\text{Regla de la cadena})$$

$$13. \frac{du}{dx} = \frac{1}{dx/du}$$

$$14. \frac{dF}{dx} = \frac{dF/du}{dx/du}$$

Derivadas de funciones exponenciales y logarítmicas

$$15. \frac{d}{dx}e^u = e^u \frac{du}{dx}$$

$$16. \frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$$17. \frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

$$18. \frac{d}{dx} \log_a u = \frac{\log_a e}{u} \frac{du}{dx} \quad a > 0, \quad a \neq 1$$

$$19. \frac{d}{dx}u^v = vu^{v-1} \frac{du}{dx} + u^v \ln u \frac{dv}{dx}$$

Derivadas de funciones trigonométricas

$$20. \frac{d}{dx} \operatorname{sen} u = \cos u \frac{du}{dx}$$

$$21. \frac{d}{dx} \operatorname{cos} u = -\operatorname{sen} u \frac{du}{dx}$$

$$22. \frac{d}{dx} \operatorname{tan} u = \sec^2 u \frac{du}{dx}$$

$$23. \frac{d}{dx} \operatorname{cot} u = -\operatorname{csc}^2 u \frac{du}{dx}$$

$$24. \frac{d}{dx} \operatorname{sec} u = \operatorname{sec} u \operatorname{tan} u \frac{du}{dx}$$

$$25. \frac{d}{dx} \operatorname{csc} u = -\operatorname{csc} u \operatorname{cot} u \frac{du}{dx}$$

Derivadas de funciones trigonométricas inversas

$$26. \frac{d}{dx} \operatorname{sen}^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad \left[-\frac{\pi}{2} < \operatorname{sen}^{-1} u < \frac{\pi}{2}\right]$$

$$27. \frac{d}{dx} \operatorname{cos}^{-1} u = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx} \quad \left[0 < \operatorname{cos}^{-1} u < \pi\right]$$

$$28. \frac{d}{dx} \operatorname{tan}^{-1} u = \frac{1}{1+u^2} \frac{du}{dx} \quad \left[-\frac{\pi}{2} < \operatorname{tan}^{-1} u < \frac{\pi}{2}\right]$$

$$29. \frac{d}{dx} \operatorname{cot}^{-1} u = \frac{-1}{1+u^2} \frac{du}{dx} \quad \left[0 < \operatorname{cot}^{-1} u < \pi\right]$$

$$30. \frac{d}{dx} \operatorname{sec}^{-1} u = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx} = \frac{\pm 1}{u\sqrt{u^2-1}} \frac{du}{dx}$$

(+) si $0 < \operatorname{sec}^{-1} u < \frac{\pi}{2}$; (-) si $\frac{\pi}{2} < \operatorname{sec}^{-1} u < \pi$

$$31. \frac{d}{dx} \csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx} = \frac{\mp 1}{u\sqrt{u^2-1}} \frac{du}{dx}$$

$$(+)\text{ si } -\frac{\pi}{2} < \csc^{-1} u < 0; \quad (-)\text{ si } 0 < \csc^{-1} u < \frac{\pi}{2}$$

Derivadas de funciones trigonométricas hiperbólicas

$$32. \frac{d}{dx} \operatorname{senh} u = \operatorname{cosh} u \frac{du}{dx}$$

$$35. \frac{d}{dx} \operatorname{coth} u = -\operatorname{csch}^2 u \frac{du}{dx}$$

$$33. \frac{d}{dx} \operatorname{cosh} u = \operatorname{senh} u \frac{du}{dx}$$

$$36. \frac{d}{dx} \operatorname{sech} u = -\operatorname{sech} u \operatorname{tanh} u \frac{du}{dx}$$

$$34. \frac{d}{dx} \operatorname{tanh} u = \operatorname{sech}^2 u \frac{du}{dx}$$

$$37. \frac{d}{dx} \operatorname{csch} u = -\operatorname{csch} u \operatorname{coth} u \frac{du}{dx}$$

Derivadas de funciones trigonométricas hiperbólicas inversas

$$38. \frac{d}{dx} \operatorname{senh}^{-1} u = \frac{1}{\sqrt{u^2+1}} \frac{du}{dx}$$

$$42. \frac{d}{dx} \operatorname{sech}^{-1} u = \frac{\pm 1}{u\sqrt{u^2-1}} \frac{du}{dx}$$

$$39. \frac{d}{dx} \operatorname{cosh}^{-1} u = \frac{\pm 1}{\sqrt{u^2-1}} \frac{du}{dx}$$

$$(+)\text{ si } \operatorname{sech}^{-1} u < 0, \quad 0 < u < 1$$

$$(-)\text{ si } \operatorname{sech}^{-1} u > 0, \quad 0 < u < 1$$

$$(+)\text{ si } \operatorname{cosh}^{-1} u > 0, \quad u > 1$$

$$(-)\text{ si } \operatorname{cosh}^{-1} u < 0, \quad u < 1$$

$$40. \frac{d}{dx} \operatorname{tanh}^{-1} u = \frac{1}{1-u^2} \frac{du}{dx} \quad [-1 < u < 1]$$

$$43. \frac{d}{dx} \operatorname{csch}^{-1} u = \frac{-1}{|u|\sqrt{1+u^2}} \frac{du}{dx} = \frac{\mp 1}{u\sqrt{1+u^2}} \frac{du}{dx}$$

$$(+)\text{ si } u < 0; \quad (-)\text{ si } u > 0$$

$$41. \frac{d}{dx} \operatorname{coth}^{-1} u = \frac{1}{1-u^2} \frac{du}{dx} \quad [u > 1 \text{ o } u < -1]$$

Integrales básicas

$$1. \int du = u + C$$

$$12. \int \operatorname{csc} u \cot u \, du = -\operatorname{csc} u + C$$

$$2. \int u \, dv = uv - \int v \, du \quad (\text{integración por partes})$$

$$13. \int \tan u \, du = \ln|\sec u| + C$$

$$3. \int u^n \, du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1$$

$$14. \int \cot u \, du = \ln|\operatorname{sen} u| + C$$

$$4. \int \frac{du}{u} = \ln|u| + C$$

$$15. \int \operatorname{sec} u \, du = \ln|\operatorname{sec} u + \operatorname{tanh} u| + C$$

$$5. \int e^u \, du = e^u + C$$

$$16. \int \operatorname{csc} u \, du = \ln|\operatorname{csc} u - \cot u| + C$$

$$6. \int a^u \, du = \frac{a^u}{\ln a} + C$$

$$17. \int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$7. \int \operatorname{sen} u \, du = -\operatorname{cos} u + C$$

$$18. \int \frac{du}{a^2 + u^2} = \frac{1}{a} \operatorname{tan}^{-1} \frac{u}{a} + C$$

$$8. \int \operatorname{cos} u \, du = \operatorname{sen} u + C$$

$$19. \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{sec}^{-1} \frac{u}{a} + C$$

$$9. \int \operatorname{sec}^2 u \, du = \operatorname{tan} u + C$$

$$20. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$$

$$10. \int \operatorname{csc}^2 u \, du = -\operatorname{cot} u + C$$

$$21. \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$$

$$11. \int \operatorname{sec} u \operatorname{tan} u \, du = \operatorname{sec} u + C$$

Integrales con funciones exponenciales y logarítmicas

$$22. \int ue^{au} du = \frac{1}{a^2}(au-1)e^{au} + C$$

$$23. \int u^n e^{au} du = \frac{1}{a}u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$24. \int \ln u du = u \ln u - u + C$$

$$25. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$26. \int \frac{1}{u \ln u} du = \ln |\ln u| + C$$

$$27. \int e^{au} \operatorname{sen} bu du = \frac{e^{au}}{a^2 + b^2} (a \operatorname{sen} bu - b \cos bu) + C$$

$$28. \int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \operatorname{sen} bu) + C$$

Integrales con funciones trigonométricas

$$29. \int u \operatorname{sen} u du = \operatorname{sen} u - u \cos u + C$$

$$30. \int u \cos u du = \cos u + u \operatorname{sen} u + C$$

$$31. \int u^n \operatorname{sen} u du = u^n \cos u + n \int u^{n-1} \cos u du$$

$$32. \int u^n \cos u du = u^n \operatorname{sen} u - n \int u^{n-1} \operatorname{sen} u du$$

$$33. \int \operatorname{sen} au \operatorname{sen} bu du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$34. \int \operatorname{sen} au \cos bu du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$35. \int \cos au \cos bu du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$36. \int \operatorname{sen}^n u \cos^m u du$$

$$= -\frac{\operatorname{sen}^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \operatorname{sen}^{n-2} u \cos^m u du$$

$$= -\frac{\operatorname{sen}^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \operatorname{sen}^n u \cos^{m-2} u du$$

$$37. \int \operatorname{sen}^2 u du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen} 2u + C$$

$$38. \int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u + C$$

$$39. \int \tan^2 u du = \tan u - u + C$$

$$40. \int \cot^2 u du = -\cot u - u + C$$

$$41. \int \operatorname{sen}^3 u du = -\frac{1}{3}(2 + \operatorname{sen}^2 u) \cos u + C$$

$$42. \int \cos^3 u du = \frac{1}{3}(2 + \cos^2 u) \operatorname{sen} u + C$$

$$43. \int \tan^3 u du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$$

$$44. \int \cot^3 u du = -\frac{1}{2} \cot^2 u - \ln |\operatorname{sen} u| + C$$

$$45. \int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

$$46. \int \csc^3 u du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

$$47. \int \operatorname{sen}^n u du = -\frac{1}{n} \operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n} \int \operatorname{sen}^{n-2} u du$$

$$48. \int \cos^n u du = \frac{1}{n} \cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n} \int \cos^{n-2} u du$$

$$49. \int \tan^n u du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u du$$

$$50. \int \cot^n u du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u du$$

$$51. \int \sec^n u du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u du$$

$$52. \int \csc^n u du = \frac{1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u du$$

Integrales con funciones trigonométricas inversas

$$53. \int \operatorname{sen}^{-1} u du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$$

$$54. \int \cos^{-1} u du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

$$55. \int \tan^{-1} u du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$$

$$56. \int u \operatorname{sen}^{-1} u du = \frac{2u^2-1}{4} \operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$57. \int u \cos^{-1} u du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$58. \int u \tan^{-1} u du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$$

$$59. \int u^n \operatorname{sen}^{-1} u du = \frac{1}{n+1} \left[u^{n+1} \operatorname{sen}^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right]$$

$$60. \int u^n \cos^{-1} u du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right]$$

$$61. \int u^n \tan^{-1} u du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1+u^2}} \right]$$

Integrales con funciones trigonométricas hiperbólicas

$$62. \int \sinh u du = \cosh u + C$$

$$67. \int \operatorname{csch} u du = \ln |\tan \frac{1}{2} u| + C$$

$$63. \int \cosh u du = \sinh u + C$$

$$68. \int \operatorname{sech}^2 u du = \tanh u + C$$

$$64. \int \tanh u du = \ln(\cosh u) + C$$

$$69. \int \operatorname{csch}^2 u du = -\operatorname{coth} u + C$$

$$65. \int \operatorname{coth} u du = \ln |\sinh u| + C$$

$$70. \int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$66. \int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$$

$$71. \int \operatorname{csch} u \operatorname{coth} u du = -\operatorname{csch} u + C$$

Integrales con $a + bu$

$$72. \int \frac{u du}{a+bu} = \frac{1}{b^2} (a+bu - a \ln |a+bu|) + C$$

$$76. \int \frac{u du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b} \ln |a+bu| + C$$

$$73. \int \frac{u^2 du}{a+bu} = \frac{1}{2b^3} \left[(a+bu)^2 - 4a(a+bu) + 2a^2 \ln |a+bu| \right] + C$$

$$77. \int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

$$74. \int \frac{du}{u(a+bu)} = \frac{1}{a} \ln \left| \frac{u}{a+bu} \right| + C$$

$$78. \int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^3} \left(a+bu - \frac{a^2}{a+bu} - 2a \ln |a+bu| \right) + C$$

$$75. \int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

Integrales con $\sqrt{a+bu}$

$$79. \int u \sqrt{a+bu} du = \frac{2}{15b^2} (3bu-2a)(a+bu)^{3/2} + C$$

$$84. \int \frac{\sqrt{a+bu}}{u^2} du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a+bu}}$$

$$80. \int \frac{u du}{\sqrt{a+bu}} = \frac{2}{3b^2} (bu-2a)\sqrt{a+bu} + C$$

$$85. \int u^n \sqrt{a+bu} du = \frac{2}{b(2n+3)} \left[u^n (a+bu)^{3/2} - na \int u^{n-1} \sqrt{a+bu} du \right]$$

$$81. \int \frac{u^2 du}{\sqrt{a+bu}} = \frac{2}{15b^3} (8a^2 + 3b^2 u^2 - 4abu)\sqrt{a+bu} + C$$

$$86. \int \frac{u^n du}{\sqrt{a+bu}} = \frac{2u^n \sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{u^{n-1} du}{\sqrt{a+bu}}$$

$$82. \int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C \quad \text{si } a > 0$$

$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bu}{-a}} + C \quad \text{si } a < 0$$

$$87. \int \frac{du}{u^n \sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a+bu}}$$

$$83. \int \frac{\sqrt{a+bu}}{u} du = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$$

Integrales con $\sqrt{2au-u^2}$

$$88. \int \sqrt{2au-u^2} du = \frac{u-a}{2} \sqrt{2au-u^2} + \frac{a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$89. \int u \sqrt{2au-u^2} du = \frac{2u-au-3a^2}{6} \sqrt{2au-u^2} + \frac{a^3}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$90. \int \frac{\sqrt{2au-u^2}}{u^2} du = \sqrt{2au-u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$91. \int \frac{\sqrt{2au-u^2}}{u^2} du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$92. \int \frac{u^2 du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2} \sqrt{2au-u^2} + \frac{3a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$93. \int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$94. \int \frac{u du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$95. \int \frac{du}{u \sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$

Integrales con $\sqrt{a^2+u^2}$

$$96. \int \sqrt{a^2+u^2} du = \frac{u}{2} \sqrt{a^2+u^2} + \frac{a^2}{2} \ln |u + \sqrt{a^2+u^2}| + C$$

$$97. \int u^2 \sqrt{a^2+u^2} du = \frac{u}{8} (a^2+2u^2) \sqrt{a^2+u^2} - \frac{a^2}{8} \ln |u + \sqrt{a^2+u^2}| + C$$

$$98. \int \frac{\sqrt{a^2+u^2}}{u} du = \sqrt{a^2+u^2} - a \ln \left| \frac{a + \sqrt{a^2+u^2}}{u} \right| + C$$

$$99. \int \frac{\sqrt{a^2+u^2}}{u^2} du = -\frac{\sqrt{a^2+u^2}}{u} + \ln |u + \sqrt{a^2+u^2}| + C$$

$$100. \int \frac{du}{\sqrt{a^2+u^2}} = \ln |u + \sqrt{a^2+u^2}| + C$$

$$101. \int \frac{u^2 du}{\sqrt{a^2+u^2}} = \frac{u}{2} \sqrt{a^2+u^2} - \frac{a^2}{2} \ln |u + \sqrt{a^2+u^2}| + C$$

$$102. \int \frac{du}{u \sqrt{a^2+u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2+u^2} + a}{u} \right| + C$$

$$103. \int \frac{du}{u^2 \sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2 u} + C$$

$$104. \int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2+u^2}} + C$$

Integrales con $\sqrt{a^2-u^2}$

$$105. \int \frac{\sqrt{a^2-u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2-u^2} - \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$106. \int \frac{u^2 du}{\sqrt{a^2-u^2}} = -\frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$107. \int \frac{du}{u \sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2-u^2}}{u} \right| + C$$

$$108. \int \frac{du}{u^2 \sqrt{a^2-u^2}} = -\frac{1}{a^2 u} \sqrt{a^2-u^2} + C$$

$$109. \int (a^2-u^2)^{3/2} du = -\frac{u}{8} (2u^2-5a^2) \sqrt{a^2-u^2} + \frac{3a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$110. \int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2-u^2}} + C$$

$$111. \int \sqrt{a^2-u^2} du = \frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$112. \int u^2 \sqrt{a^2-u^2} du = \frac{u}{8} (2u^2-a^2) \sqrt{a^2-u^2} + \frac{a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$113. \int \frac{\sqrt{a^2-u^2}}{u} du = \sqrt{a^2-u^2} - a \ln \left| \frac{a + \sqrt{a^2-u^2}}{u} \right| + C$$

Integrales con $\sqrt{u^2 - a^2}$

$$114. \int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln|u + \sqrt{u^2 - a^2}| + C$$

$$115. \int u^2 \sqrt{u^2 - a^2} du \\ = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln|u + \sqrt{u^2 - a^2}| + C$$

$$116. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{u} + C$$

$$117. \int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln|u + \sqrt{u^2 - a^2}| + C$$

$$118. \int \frac{du}{\sqrt{u^2 - a^2}} = \ln|u + \sqrt{u^2 - a^2}| + C$$

$$119. \int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln|u + \sqrt{u^2 - a^2}| + C$$

$$120. \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$121. \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$