ECUACIÓN DE CONSERVACIÓN DE MOMENTUM

en función del esfuerzo cortante (para cualquier tipo de fluido)

$$\underbrace{\rho\frac{\partial \mathbf{v}}{\partial t}}_{\text{acumulación}} \underbrace{+\rho\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{advección}} \underbrace{+\nabla\cdot\boldsymbol{\tau}}_{\text{transporte viscoso}} \underbrace{+\nabla P}_{\text{presión}} \underbrace{-\rho\mathbf{g}}_{\text{gravedad}} = 0$$

Coordenadas Rectangulares

componente	x	$\rho\frac{\partial v_x}{\partial t}+\rho\left[v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right]+\left[\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right]+\frac{\partial P}{\partial x}-\rho g_x=0$
	y	$\rho\frac{\partial v_y}{\partial t}+\rho\left[v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right]+\left[\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right]+\frac{\partial P}{\partial y}-\rho g_y=0$
	z	$\rho\frac{\partial v_z}{\partial t}+\rho\left[v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right]+\left[\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right]+\frac{\partial P}{\partial z}-\rho g_z=0$

Coordenadas Cilíndricas

componente	r	$\rho\frac{\partial v_r}{\partial t}+\rho\left[v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^{2}}{r}+v_z\frac{\partial v_r}{\partial z}\right]+\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)+\frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta}-\frac{\tau_{\theta\theta}}{r}+\frac{\partial \tau_{zr}}{\partial z}\right]+\frac{\partial P}{\partial r}-\rho g_r=0$
	θ	$\rho\frac{\partial v_\theta}{\partial t}+\rho\left[v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right]+\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right)+\frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta}+\frac{\partial \tau_{z\theta}}{\partial z}\right]+\frac{1}{r}\frac{\partial P}{\partial \theta}-\rho g_\theta=0$
	z	$\rho\frac{\partial v_z}{\partial t}+\rho\left[v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right]+\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)+\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}+\frac{\partial \tau_{zz}}{\partial z}\right]+\frac{\partial P}{\partial z}-\rho g_z=0$

Coordenadas Esféricas

componente	r	$\rho\frac{\partial v_r}{\partial t}+\rho\left[v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\phi}{r\operatorname{sen}\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^{2}+v_\phi^{2}}{r}\right]$ $+\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{rr}\right)+\frac{1}{r\operatorname{sen}\theta}\frac{\partial}{\partial \theta}\left(\tau_{\theta r}\operatorname{sen}\theta\right)+\frac{1}{r\operatorname{sen}\theta}\frac{\partial \tau_{\phi r}}{\partial \phi}-\frac{\tau_{\theta\theta}+\tau_{\phi\phi}}{r}\right]+\frac{\partial P}{\partial r}-\rho g_r=0$
	θ	$\rho\frac{\partial v_\theta}{\partial t}+\rho\left[v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}-\frac{v_\phi}{r\operatorname{sen}\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^{2}\cot\theta}{r}\right]$ $+\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right)+\frac{1}{r\operatorname{sen}\theta}\frac{\partial}{\partial \theta}\left(\tau_{\theta\theta}\operatorname{sen}\theta\right)+\frac{1}{r\operatorname{sen}\theta}\frac{\partial \tau_{\phi\theta}}{\partial \phi}+\frac{\tau_{r\theta}-\tau_{\phi\phi}\cot\theta}{r}\right]+\frac{1}{r}\frac{\partial P}{\partial \theta}-\rho g_\theta=0$
	ϕ	$\rho\frac{\partial v_\phi}{\partial t}+\rho\left[v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}-\frac{v_\phi}{r\operatorname{sen}\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r v_\phi}{r}-\frac{v_\theta v_\phi\cot\theta}{r}\right]$ $+\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\phi}\right)+\frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta}+\frac{1}{r\operatorname{sen}\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi}+\frac{\tau_{r\phi}-2\tau_{\theta\phi}\cot\theta}{r}\right]+\frac{1}{r\operatorname{sen}\theta}\frac{\partial P}{\partial \phi}-\rho g_\phi=0$

ECUACIÓN DE CONSERVACIÓN DE MOMENTUM (Navier-Stokes)

en función del gradiente de velocidad (para fluido newtoniano de viscosidad constante)

$$\underset{\text{acumulación}}{\rho\frac{\partial \mathbf{v}}{\partial t}} \quad \underset{\text{advección}}{+\rho\mathbf{v}\cdot\nabla\mathbf{v}} \quad \underset{\text{transporte viscoso}}{-\mu\nabla^2\mathbf{v}} \quad \underset{\text{presión}}{+\nabla P} \quad \underset{\text{gravedad}}{-\rho\mathbf{g}} \quad = 0$$

Coordenadas Rectangulares

componente		
componente	x	$\rho\frac{\partial v_x}{\partial t}+\rho\left[v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right]-\mu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]+\frac{\partial P}{\partial x}-\rho g_x=0$
	y	$\rho\frac{\partial v_y}{\partial t}+\rho\left[v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right]-\mu\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\frac{\partial P}{\partial y}-\rho g_y=0$
	z	$\rho\frac{\partial v_z}{\partial t}+\rho\left[v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right]-\mu\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\frac{\partial P}{\partial z}-\rho g_z=0$

Coordenadas Cilíndricas

componente		
componente	r	$\rho\frac{\partial v_r}{\partial t}+\rho\left[v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^{\,2}}{r}+v_z\frac{\partial v_r}{\partial z}\right]-\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\frac{\partial P}{\partial r}-\rho g_r=0$
	θ	$\rho\frac{\partial v_\theta}{\partial t}+\rho\left[v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right]-\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$ $+\frac{1}{r}\frac{\partial P}{\partial\theta}-\rho g_\theta=0$
	z	$\rho\frac{\partial v_z}{\partial t}+\rho\left[v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right]-\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\frac{\partial P}{\partial z}-\rho g_z=0$

Coordenadas Esféricas

componente		
componente	r	$\rho\frac{\partial v_r}{\partial t}+\rho\left[v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+\frac{v_\phi}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial\phi}-\frac{v_\theta^{\,2}+v_\phi^{\,2}}{r}\right]-\frac{\mu}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial v_r}{\partial r}\right)+\frac{1}{\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial v_r}{\partial\theta}\right)\right.$ $\left.+\frac{1}{\mathrm{sen}^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}-2v_r-2v_\theta\cot\theta-2\frac{\partial v_\theta}{\partial\theta}-\frac{2}{\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial\phi}\right]+\frac{\partial P}{\partial r}-\rho g_r=0$
	θ	$\rho\frac{\partial v_\theta}{\partial t}+\rho\left[v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_\phi}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial\phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^{\,2}\cot\theta}{r}\right]-\frac{\mu}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial v_\theta}{\partial\theta}\right)\right.$ $\left.+\frac{1}{\mathrm{sen}^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2}+2\frac{\partial v_r}{\partial\theta}-\frac{v_\theta}{\mathrm{sen}^2\theta}-\frac{2\cos\theta}{\mathrm{sen}^2\theta}\frac{\partial v_\phi}{\partial\phi}\right]+\frac{1}{r}\frac{\partial P}{\partial\theta}-\rho g_\theta=0$
	ϕ	$\rho\frac{\partial v_\phi}{\partial t}+\rho\left[v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial\theta}+\frac{v_\phi}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial\phi}+\frac{v_r v_\phi}{r}-\frac{v_\theta v_\phi\cot\theta}{r}\right]-\frac{\mu}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)+\frac{1}{\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial v_\phi}{\partial\theta}\right)\right.$ $\left.+\frac{1}{\mathrm{sen}^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2}-\frac{v_\phi}{\mathrm{sen}^2\theta}+\frac{2}{\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial\phi}+\frac{2\cos\theta}{\mathrm{sen}^2\theta}\frac{\partial v_\theta}{\partial\phi}\right]+\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial P}{\partial\phi}-\rho g_\phi=0$