

ECUACIÓN DE CONSERVACIÓN DE MOMENTUM

en función del esfuerzo cortante (para cualquier tipo de fluido)

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} + \nabla \cdot \boldsymbol{\tau} + \nabla P - \rho \mathbf{g} = 0$$

acumulación advección transporte viscoso presión gravedad

Coordenadas Rectangulares

componente	X	$\rho \frac{\partial v_x}{\partial t} + \rho \left[v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right] + \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right] + \frac{\partial P}{\partial x} - \rho g_x = 0$
componente	y	$\rho \frac{\partial v_y}{\partial t} + \rho \left[v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right] + \left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right] + \frac{\partial P}{\partial y} - \rho g_y = 0$
componente	Z	$\rho \frac{\partial v_z}{\partial t} + \rho \left[v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right] + \left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right] + \frac{\partial P}{\partial z} - \rho g_z = 0$

Coordenadas Cilíndricas

componente	r	$\rho \frac{\partial v_r}{\partial t} + \rho \left[v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right] + \left[\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) + \frac{1}{r} \frac{\partial \tau_{\theta r}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{zr}}{\partial z} \right] + \frac{\partial P}{\partial r} - \rho g_r = 0$
componente	\theta	$\rho \frac{\partial v_\theta}{\partial t} + \rho \left[v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right] + \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\theta}) + \frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{z\theta}}{\partial z} \right] + \frac{1}{r} \frac{\partial P}{\partial \theta} - \rho g_\theta = 0$
componente	Z	$\rho \frac{\partial v_z}{\partial t} + \rho \left[v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right] + \left[\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} \right] + \frac{\partial P}{\partial z} - \rho g_z = 0$

Coordenadas Esféricas

componente	r	$\rho \frac{\partial v_r}{\partial t} + \rho \left[v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right] + \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{rr}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\tau_{\theta r} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial \tau_{\phi r}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r} \right] + \frac{\partial P}{\partial r} - \rho g_r = 0$
componente	\theta	$\rho \frac{\partial v_\theta}{\partial t} + \rho \left[v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} - \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right] + \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\theta}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\tau_{\theta\theta} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial \tau_{\phi\theta}}{\partial \phi} + \frac{\tau_{r\theta} - \tau_{\phi\phi} \cot \theta}{r} \right] + \frac{1}{r} \frac{\partial P}{\partial \theta} - \rho g_\theta = 0$
componente	\phi	$\rho \frac{\partial v_\phi}{\partial t} + \rho \left[v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} - \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} - \frac{v_\theta v_\phi \cot \theta}{r} \right] + \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\phi}) + \frac{1}{r} \frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\phi} - 2\tau_{\theta\phi} \cot \theta}{r} \right] + \frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} - \rho g_\phi = 0$

ECUACIÓN DE CONSERVACIÓN DE MOMENTUM (Navier-Stokes)

en función del gradiente de velocidad (para fluido newtoniano de viscosidad constante)

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} - \mu \nabla^2 \mathbf{v} + \nabla P - \rho \mathbf{g} = 0$$

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Coordenadas Rectangulares

componente	X	$\rho \frac{\partial v_x}{\partial t} + \rho \left[v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right] - \mu \left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] + \frac{\partial P}{\partial x} - \rho g_x = 0$
	Y	$\rho \frac{\partial v_y}{\partial t} + \rho \left[v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right] - \mu \left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right] + \frac{\partial P}{\partial y} - \rho g_y = 0$
	Z	$\rho \frac{\partial v_z}{\partial t} + \rho \left[v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right] - \mu \left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \frac{\partial P}{\partial z} - \rho g_z = 0$

Coordenadas Cilíndricas

componente	r	$\rho \frac{\partial v_r}{\partial t} + \rho \left[v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right] - \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] + \frac{\partial P}{\partial r} - \rho g_r = 0$
	θ	$\rho \frac{\partial v_\theta}{\partial t} + \rho \left[v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right] - \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] + \frac{1}{r} \frac{\partial P}{\partial \theta} - \rho g_\theta = 0$
	Z	$\rho \frac{\partial v_z}{\partial t} + \rho \left[v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right] - \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \frac{\partial P}{\partial z} - \rho g_z = 0$

Coordenadas Esféricas

componente	r	$\rho \frac{\partial v_r}{\partial t} + \rho \left[v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right] - \frac{\mu}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial v_r}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_r}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 v_r}{\partial \phi^2} - 2v_r - 2v_\theta \cot \theta - 2 \frac{\partial v_\theta}{\partial \theta} - \frac{2}{\sin \theta} \frac{\partial v_\phi}{\partial \phi} \right] + \frac{\partial P}{\partial r} - \rho g_r = 0$
	θ	$\rho \frac{\partial v_\theta}{\partial t} + \rho \left[v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right] - \frac{\mu}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\theta}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_\theta}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 v_\theta}{\partial \phi^2} + 2 \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{\sin^2 \theta} - \frac{2 \cos \theta}{\sin^2 \theta} \frac{\partial v_\phi}{\partial \phi} \right] + \frac{1}{r} \frac{\partial P}{\partial \theta} - \rho g_\theta = 0$
	ϕ	$\rho \frac{\partial v_\phi}{\partial t} + \rho \left[v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} - \frac{v_\theta v_\phi \cot \theta}{r} \right] - \frac{\mu}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial v_\phi}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v_\phi}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 v_\phi}{\partial \phi^2} - \frac{v_\phi}{\sin^2 \theta} + \frac{2}{\sin \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cos \theta}{\sin^2 \theta} \frac{\partial v_\theta}{\partial \phi} \right] + \frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} - \rho g_\phi = 0$